FAR BEYOND

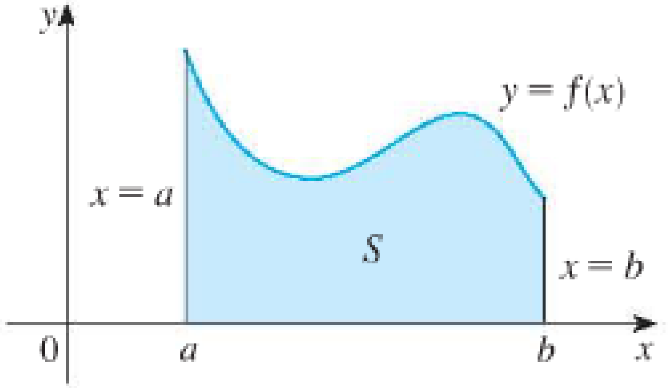
MAT122

Area Under Curve – part II

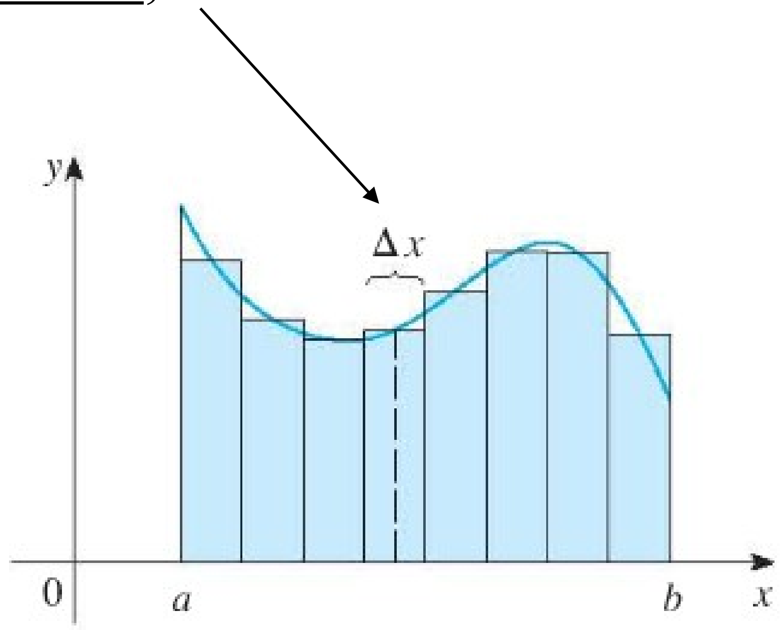
Stony Brook University

Area Under a Curve – Non-standard Shape - recall

Riemann Sum is used to *estimate* the area under a curve using a series of either left- or right-handed rectangles.



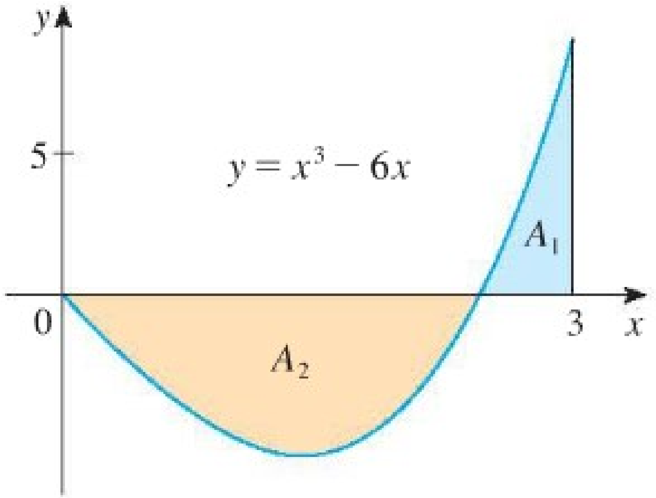
All rectangles have the same base, Δx.



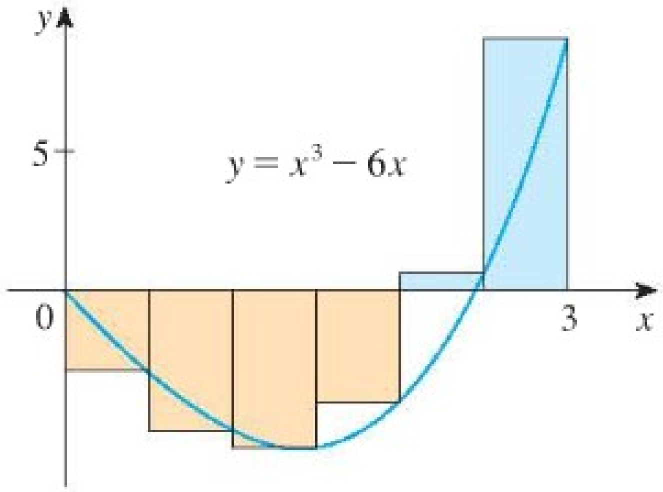
Accuracy of Riemann Sum – recall

The accuracy of the estimation will increase when more boxes are used.

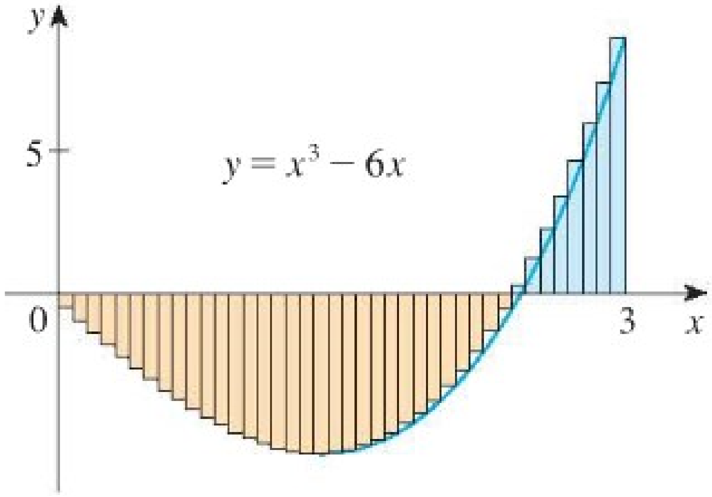
When finding the estimate of this area:



Accuracy using these boxes will be low…



… while accuracy using these will be high.



Left-Hand and Right-Hand Riemann Sum Formula

recall: heights of boxes are determined by y-values of the function

Left hand Riemann Sum:

$$A \approx L_n = \Delta x(f(x_0) + f(x_1) + \ldots + f(x_{n-1}))$$

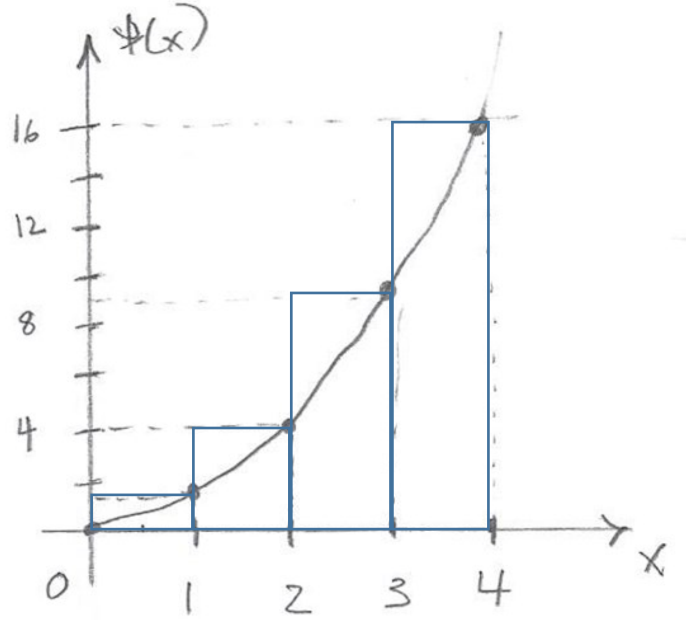
Right hand Riemann Sum:

$$A \approx R_n = \Delta x(f(x_1) + f(x_2) + \ldots + f(x_n))$$

Evaluating Using Right-Hand Sum

ex. Estimate the area under the graph $f(x) = x^2$ from $x = 0$ to $x = 4$
*using **four** approximating rectangles and **right** endpoints.*

$$R_n = \Delta x(f(x_1) + f(x_2) + \ldots + f(x_n))$$



To calculate Δx:

$$\Delta x = \frac{b-a}{n}$$

number of rectangles

$$\Delta x = \frac{4-0}{4} = 1$$

$$R_4 = \Delta x(f(x_1) + f(x_2) + f(x_3) + f(x_4))$$

$$= 1(f(1) + f(2) + f(3) + f(4))$$

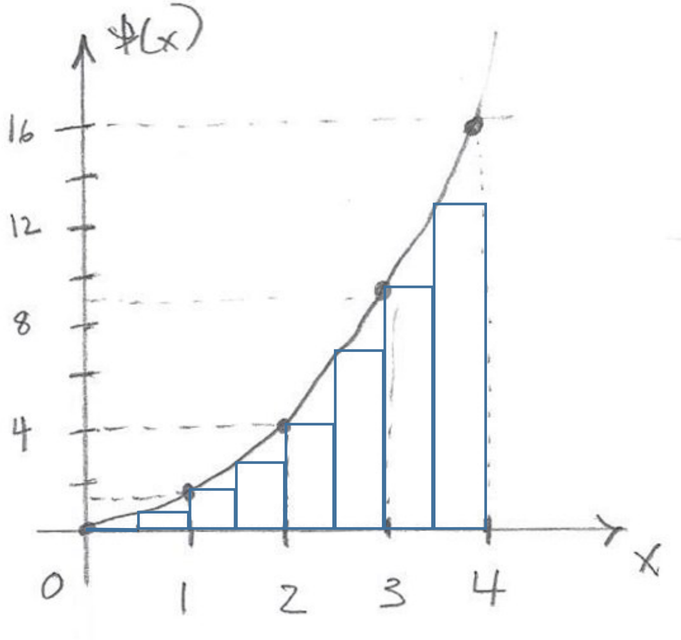
$$= 1^2 + 2^2 + 3^2 + 4^2$$

$$= 1 + 4 + 9 + 16 = \boxed{30}$$

Evaluating Using Left-Hand Sum

ex. Estimate the area under the graph $f(x) = x^2$ from $x = 0$ to $x = 4$
*using **eight** approximating rectangles and **left** endpoints.*

$$L_n = \Delta x(f(x_0) + f(x_1) + \ldots + f(x_{n-1}))$$



$$\Delta x = \frac{b-a}{n}$$

number of rectangles

$$\Delta x = \frac{4-0}{8} = 1/2$$

$$L_8 = \Delta x(f(x_0) + f(x_1) + f(x_2) + f(x_3) + f(x_4) + f(x_5) + f(x_6) + f(x_7))$$

$$= \frac{1}{2}\left(f(0) + f\left(\frac{1}{2}\right) + f(1) + f\left(\frac{3}{2}\right) + f(2) + f\left(\frac{5}{2}\right) + f(3) + f\left(\frac{7}{2}\right)\right)$$

$$= \frac{1}{2}\left(0^2 + \left(\frac{1}{2}\right)^2 + 1^2 + \left(\frac{3}{2}\right)^2 + 2^2 + \left(\frac{5}{2}\right)^2 + 3^2 + \left(\frac{7}{2}\right)^2\right) = \frac{1}{2}(35) = \frac{35}{2} = \boxed{17.5}$$

Right- and Left-Hand Sum – Example #2

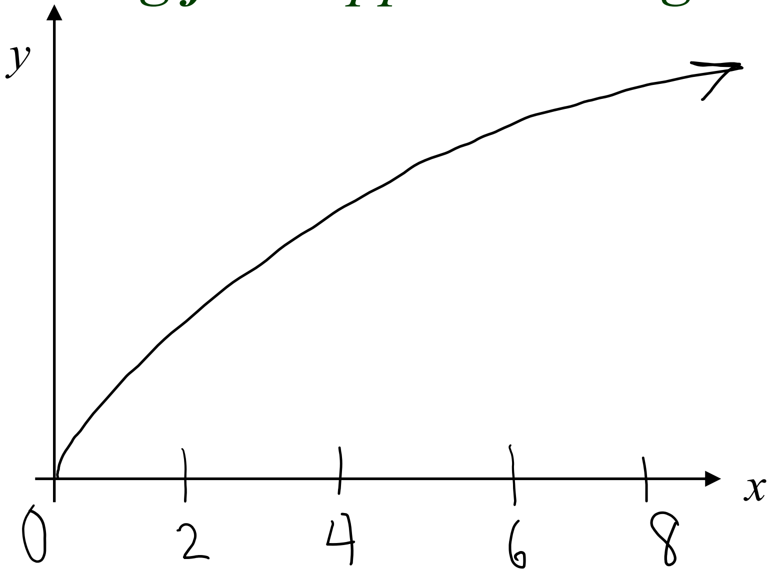
ex. Estimate the area under the graph $f(x)=\sqrt{x}$ from $x=0$ to $x=8$

*using **four** approximating rectangles and…*

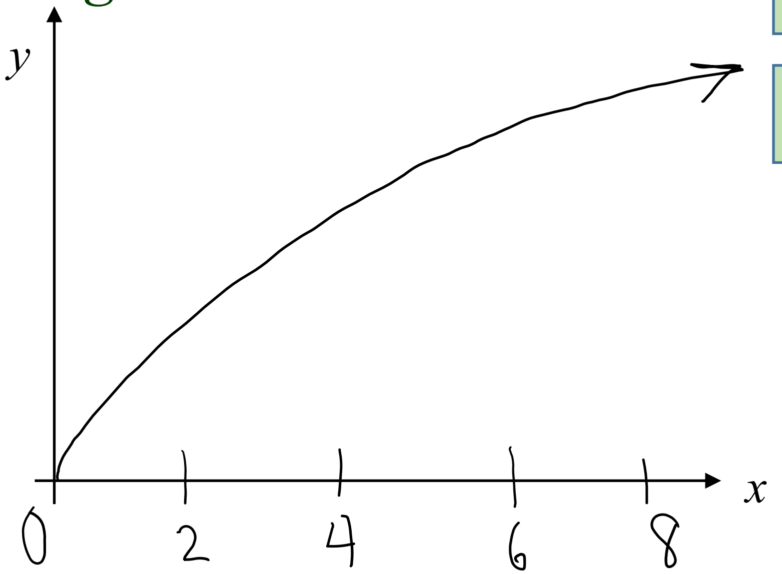
$$R_n = \Delta x(f(x_1)+f(x_2)+\ldots+f(x_n))$$

$$L_n = \Delta x(f(x_0)+f(x_1)+\ldots+f(x_{n-1}))$$

$$\Delta x = \frac{8-0}{4} = 2 \qquad \Delta x = \frac{b-a}{n}$$



*…**right** endpoints.*



*…**left** endpoints.*

$$R_4 = \Delta x(f(2)+f(4)+f(6)+f(8))$$
$$= 2(\sqrt{2}+\sqrt{4}+\sqrt{6}+\sqrt{8})$$
$$= 2(1.4+2+2.5+2.8)$$
$$= 2(8.7) = \boxed{17.4}$$

$$L_4 = \Delta x(f(0)+f(2)+f(4)+f(6))$$
$$= 2(0+\sqrt{2}+\sqrt{4}+\sqrt{6})$$
$$= 2(0+1.4+2+2.5)$$
$$= 2(5.9) = \boxed{11.8}$$